

Algebra qualifying exam, Spring 2016

1. Classify the groups of order 24 having trivial center.
2. Let  $V$  be a finite dimensional vector space over a field of characteristic  $p$  and let  $T : V \rightarrow V$  be a linear transformation such that  $T^p = I$

$\bar{d}$ ) for some  $d \in \mathbb{Z}$ .

5. Suppose  $n$  and  $m$  are positive integers, let  $R = \mathbb{Z}[X]/(X^n)$  and let  $M$  be an  $R$ -module, let  $x$  be the image of  $X$  in  $R$ , and let  $(x^m)$  be the ideal in  $R$  generated by  $x^m$ . Compute  $\text{Tor}_i^R(M, (x^m))$  for all  $i$ .
6. Let  $k$  be a field. Find the minimal primes and compute the Krull dimension of  $k[x, y, z]/(xy, xz)$ .
7. Let  $R$  be an Artinian local ring. Prove that an  $R$ -module is flat if and only if it is free.
8. Suppose that  $R$  is a Noetherian ring and  $\mathfrak{p}$  is a prime ideal such that  $R_{\mathfrak{p}}$  is an integral domain. Show that there is an  $f \in R \setminus \mathfrak{p}$  such that  $R_f$  is an integral domain. (Recall that  $R_f = S^{-1}R$  where  $S = \{1, f, f^2, f^3, \dots\}$ .)